Quiz 8
Math 31–6429

You should fully justify your answers. Do all your work on separate paper, and make sure to *print* your name in the first sheet and staple all the sheets together. **Unstapled, loose pieces of paper will not be graded.** This quiz is due on Thursday, October 25, at 6:00 pm.

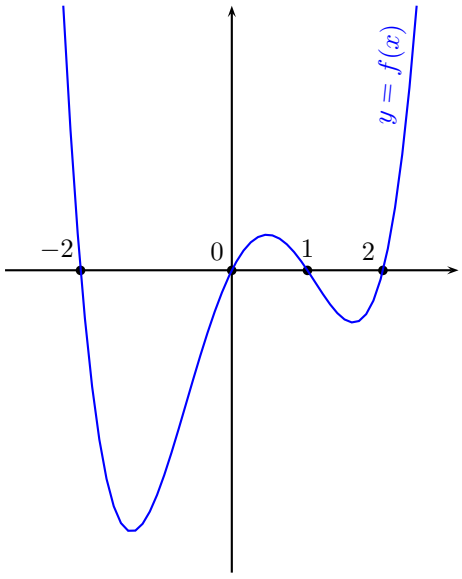
1. Find $f'(x)$ and $g'(x)$ where $f(x) = \displaystyle\int_1^x \frac{\sin t^2 + 3}{\cos(3t+5)}\,dt$ and $g(x) = \displaystyle\int_1^{x^3} \frac{\sin t^2 + 3}{\cos(3t+5)}\,dt$

2. Calculate $\displaystyle\int_1^3 \sqrt{1-(x-2)^2}\,dx$.

3. Calculate $\displaystyle\int_{-4}^4 (x - 3x^3 + 6x^5)\cos x\,dx$.

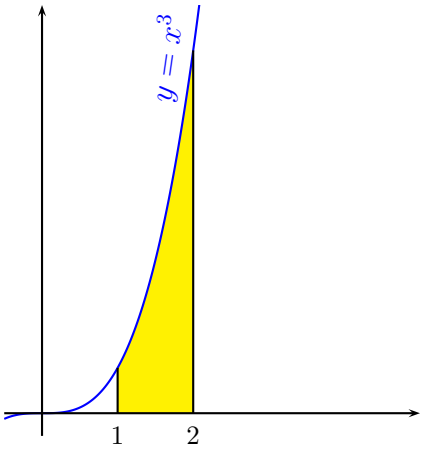
4. Calculate $\displaystyle\int_1^4 \left(x + x\sqrt{x}\right)\,dx$

5. Consider the function $f(x) = x^4 - x^3 - 4x^2 + 4x$.
   (a) Calculate $\displaystyle\int_{-2}^2 f(x)\,dx$
   (b) Find the area of the region contained between the graph of $y = f(x)$ and the x–axis between $x = -2$ and $x = 2$.



6. A particle moves in a straight line and its velocity at time t is given by the function
$$v(t) = \cos t, \qquad 0 \le t \le \pi$$
   (a) Find the displacement of the particle for the time period $0 \le t \le \pi$.
   (b) Find the distance traveled during this period.

7. **Extra Credit** Consider the following region:



(a) Partition the interval $[1, 2]$ into n equal subintervals each of legth Δx. At this point you should calculate

i. The length Δx.

ii. The right endpoint of each subinterval.

iii. The left endpoint of each subinterval.

iv. The midpoint of each subinterval.

(b) Calculate the area of the "left", "middle", and "right" rectangles supported on each subinterval.

(c) Use the formula

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

to find a formula for the left, middle and right Riemann sums.

(d) Verify that as $n \to \infty$ all these Riemann sums converge to the same number.

(e) Use the Fundamental Theorem of Calculus to verify the above calculations.