BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 31 Quiz 7
Nikos Apostolakis November 15, 2007

The Answers

1. Calculate the following limits. If a limit does not exist state so and explain why.
   (a) $\lim_{x\to\infty} \frac{3x^2-16}{5x^2+9x+20} = \frac{3}{5}$
   (b) $\lim_{x\to\infty} \frac{x^3+4x^2-5x+6}{3x^2-5x+1} = \infty$
   (c) $\lim_{x\to-\infty} (x^3-x^4) = -\infty$
   (d) $\lim_{x\to-\infty} \frac{\sqrt{4x^2-3x+1}}{2x+5} = -1$

2. Find the absolute extrema of the following function:

$$f(x) = \left|x^2+x-2\right|$$

   Answer. Critical points are at $x=-2$ and $x=1$ where the derivative does not exist and at $x=-\frac{1}{2}$ where the derivative vanishes. We have local minima at $x=1$ and $x=-2$ and a local maximum at $x=-1/2$. The corresponding values are 0, 0 and 9/4 respectively. Since $f(x)\geq 0$ the minima are absolute. On the other hand, at $x=-1/2$ the maximum is not absolute since, for example, $f(2)=4$ is a larger value. □

3. (a) Prove that the equation $x^3+33x-9x^2-65=0$ has a solution in the interval $[0,10]$.

   Proof. Let $f(x)=x^3+33x-9x^2-65$. Then f, being a polynomial, is continuous and we have: $f(0)=-65$ while $f(10)=365$. Since 0 is between these two values it follows by the Intermidiate value theorem that there is a c in the interval $(0,10)$ such that $f(c)=0$. □

   (b) Prove that the equation in part (a) has *only* one solution.

   Answer. We have $f'(x)=3x^2-18x+33=3(x^2-6x+11)$ a quadratic polynomial with discriminant $b^2-4ac=-8$. Therefore the derivative of f has no real roots. Now, if f had two (or more) roots, by Rolle's theorem f' would have at least one root. It follows that f cannot have more than two roots. □

4. Use Newton's method to estimate $\sqrt{3}$. Use three iterrations.

Answer. $\sqrt{3}$ is the positive solution of the equation $x^2 - 3 = 0$. So we will use Newtons method to find a root of the function $f(x) = x^2 - 3$. We have $f'(x) = 2x$ so that the itteration is given by

$$x_{n+1} = x_n - \frac{x_n^2 - 3}{2x_n}$$

Let's start with $x_1 = 2$. Then we have:

$$x_2 = \frac{7}{4}$$
$$x_3 = \frac{97}{56}$$

Thus we have the approximation $\sqrt{3} \approx 1.73214285714$ which is correct to the third decimal place. □

5. Consider the function:

$$f(x) = \frac{x^2}{x^2 - 4}$$

Draw a rough sketch of the graph of f. The graph should correctly indicate local extrema, assymptotes, intervals where the function is increasing or decreasing and intervals where the function is concave upward or downward.

Answer. The domain of f is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$. We have

$$\lim_{x \to -2^-} f(x) = \infty$$
$$\lim_{x \to -2^+} f(x) = -\infty$$
$$\lim_{x \to 2^-} f(x) = -\infty$$
$$\lim_{x \to 2^+} f(x) = \infty$$

Thus we have two vertical assymptotes $x = -2$ and $x = 2$. Furtermore the numerator and denominator have the same degree so we have a horizontal assymptote. We have:

$$\lim_{x \to \infty} f(x) = 1$$
$$\lim_{x \to -\infty} f(x) = 1$$

We have

$$f'(x) = \frac{-8x}{(x^2 - 4)^2}$$

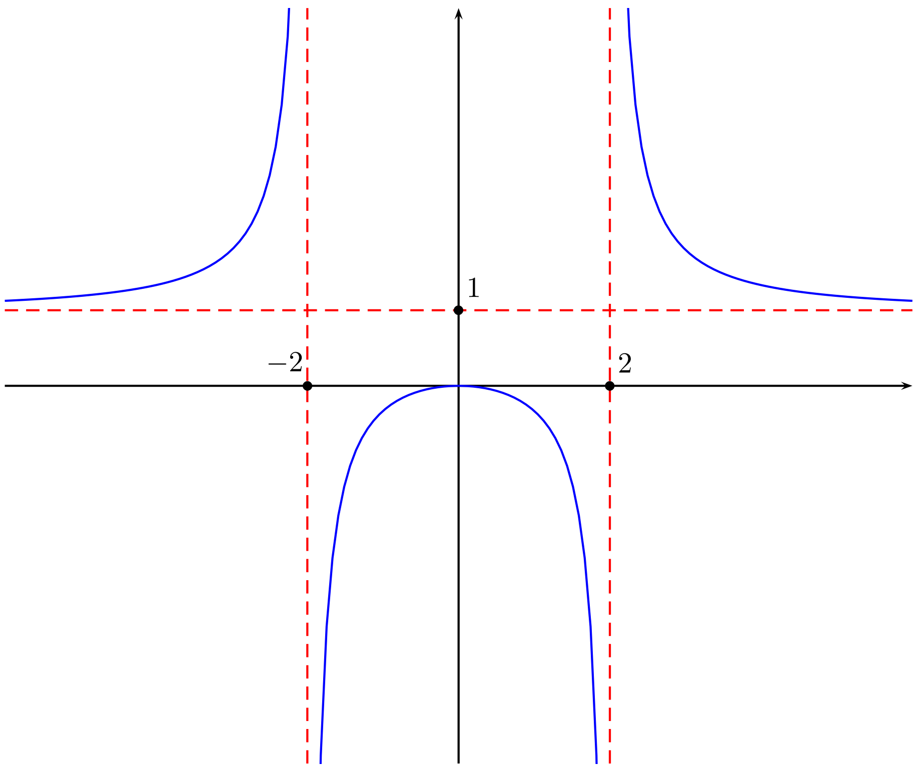
and

$$f''(x) = \frac{24x^2 + 32}{(x^2 - 4)^3}$$

There is a critical point at $x = 0$ while the second derivative has no roots. Therefore there are no inflection points. We can summarize the information about the sign of the first and second derivaive on the following table.

x	$-\infty$		-2		0		2		∞
$f'(x)$		$+$		$+$	0	$-$		$-$	
$f''(x)$		$+$		$-$		$-$		$+$	
$f(x)$					max				

So we have the following sketch:



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