

Quiz 6
Math 31–6429

You should fully justify your answers. Do all your work on separate paper, and make sure to *print* your name in the first sheet and staple all the sheets together. **Unstapled, loose pieces of paper will not be graded.** This quiz is due on Thursday, October 25, at 6:00 pm.

1. Find the *absolute* maximum and minimum of the function

$$f(x) = \frac{x}{x^2 + 1},$$

on the interval $[-3, 2]$.

2. Verify that $f(x) = x^2 - 2x - 1$ satisfy the hypothesis of Rolle's theorem in the interval $[-2, 4]$. Then find all numbers c that satisfy the conclusion of Rolle's theorem.

3. Let $f(x) = \frac{x+2}{x-2}$.

- (a) Show that there is no c such that $f(3) - f(1) = f'(c)(3 - 1)$.
(b) Why does (a) not contradict Mean Value Theorem?

4. Prove that that the equation $\cos x = x$ has exactly one solution in the interval $(0, 1)$.

Hint. You need to use two theorems. One to prove that there exists a solution and a second one to prove that there exists only one solution.

5. **Extra Credit** A number a is called a fixed point of a function f if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x then f has at most one fixed point.