

Quiz 2
Math 31–6429

You should fully justify your answers. Do all your work on separate paper, and make sure to *print* your name in the first sheet and staple all the sheets together. **Unstapled, loose pieces of paper will not be graded.** This quiz is due on Thursday, September 20, at 6:00pm.

1. Explain *in detail* why the function

$$f(x) = \frac{(2+x)^3 - 7}{1+x^2} - \sqrt{x^2 + 3} - \sin(\cos(3x))$$

is continuous at all points of its domain.

2. Find the points that each of the following functions is discontinuous and identify the nature of the discontinuity:

$$(a) f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$(b) g(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$$

$$(c) g(x) = \begin{cases} \frac{3}{(x-5)^2} & \text{if } x \neq 5 \\ 5 & \text{if } x = 5 \end{cases}$$

3. Find the real number a so that the function defined by

$$f(x) = \begin{cases} 2x - a & \text{if } -\infty \leq x \leq \pi \\ \sin x & \text{if } \pi < x < \infty \end{cases}$$

is continuous on \mathbb{R} .

4. Give an example of a function that

- (a) has a jump discontinuity at $x = -5$.
- (b) has a removable singularity at $x = 0$.
- (c) has an infinite discontinuity at $x = 3$.
- (d) is continuous everywhere except at $x = 0$ and the discontinuity is not jump, removable or infinite.

5. Prove that the equation $2^x = x^2$ has a solution in the interval $[-1, 0]$.¹

¹This equation has also two obvious positive solutions can you find them?