Review for the first midterm for Math 30

Nikos Apostolakis

1. Graph each of the following functions. Possible x or y intercepts should be identified exactly.
   (a) $f(x) = 3x - 2$
   (b) $g(x) = -2x^2 + 8x + 10$
   (c) $h(x) = \dfrac{3}{2x^3}$
   (d) $f(x) = -2\sqrt[4]{x}$
   (e) $g(x) = |x - 3| + 2$
   (f) $h(x) = 2(x-1)^3 + 2$
   (g) $h(x) = -(x-1)(2x-4)(3-x)(x+2)$
   (h) $g(x) = (2x-5)^2(x-2)^3(x+1)$
   (i) $g(x) = [\![x+2]\!]$
   (j) $f(x) = \begin{cases} -x^2 & \text{if } x < -3 \\ 2x+3 & \text{if } -3 \le x \le 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$

2. For each of the following functions find the domain and the formula for $f+g$, $f \cdot g$, $\dfrac{f}{g}$:
   (a) $f(x) = 2x$, $g(x) = x^2 - 4$
   (b) $f(x) = \sqrt{x+1}$, $g(x) = x + 3$
   (c) $f(x) = x^3 + 2$, $g(x) = \sqrt{x-3}$
   (d) $f(x) = -x^3$, $g(x) = [\![x+1]\!]$

3. For each of the following pair of fucnctions find $f \circ g$ and $g \circ f$. You should identify the domain and the formula.
   (a) $f(x) = \sqrt{3x-2}$, $g(x) = x^2$
   (b) $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{3}{x+1}$
   (c) $f(x) = \dfrac{1}{x}$, $g(x) = x - 3$
   (d) $f(x) = x^2 - 1$, $g(x) = x^2 + 2x + 1$
   (e) $f(x) = \sqrt{x}$, $g(x) = x^2$
   (f) $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{1}{x}$

4. Given the graphs of $f(x)$, $g(x)$, and $h(x)$ in Figure 1 answer the following questions for each of them:

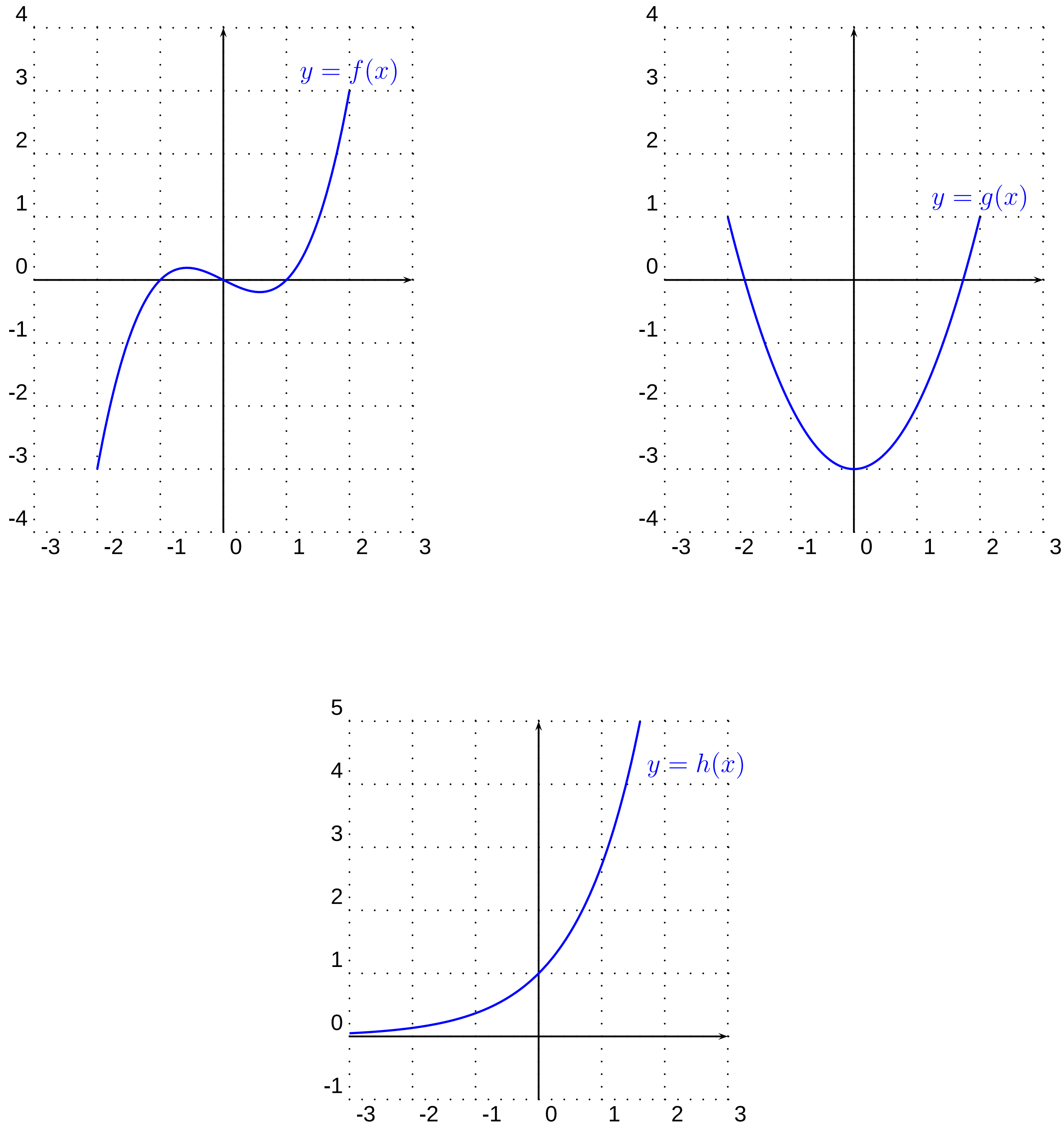

Figure 1: The graphs refered to in question 4

(a) What is the domain of this function?

(b) What is the range of this function?

(c) Over what intervals is the function increasing?

(d) Over what intervals is the function decreasing?

(e) Is the function even, odd, or neither even nor odd?

5. Is there any function that is both even *and* odd?

6. Let $f(x) = \dfrac{x+2}{x}$ and $g(x) = \dfrac{2}{x-1}$. Find $f \circ g$ and $g \circ f$. What does your result mean?

7. A ball is thrown upwards with an initial velocity of 48 ft/sec from the top of 144–foot building. What is the maximum height that the ball achieves? The height of the ball at time t is given by:
$$h(t) = -16t^2 + 48t + 144.$$

8. Of all rectangles with perimeter 40 meters find the one with the largest area.

9. The total revenue of a factory producing a certain product is given by the function $R(x) = 300x - 0.1x^2$, where x is the number of units produced. How many units must be produced so that the revenue is maximum?

10. Find the domain of each of the following functions. Express your answers using interval notation.

    (a) $f(x) = x^2 - 5x + 7$

    (b) $g(x) = \dfrac{3x}{x^2 - x - 6}$

    (c) $h(x) = \sqrt[3]{6x - 3}$

    (d) $k(x) = \sqrt{4x - 12}$

    (e) $l(x) = \dfrac{3x}{\sqrt{2 - 4x}}$

    (f) $g(x) = \sqrt{-2x}$

11. For each of the following functions find the domain, the range and the inverse function:

    (a) $g(x) = -x$

    (b) $f(x) = 4x + 5$

    (c) $f(x) = \dfrac{1}{x}$

    (d) $f(x) = \dfrac{1}{x-1}$

    (e) $g(x) = \dfrac{x-2}{3x+1}$

(f) $f(x) = x^3 + 4$

(g) $h(x) = 5\sqrt[3]{4x}$

(h) $g(x) = -\sqrt{-x}$

12. Consider the function: $f(x) = 2x^2 - 12x + 16$.

    (a) Prove that f is not a 1–1 function.

    (b) How can we restrict the domain of f so that it becomes 1–1?

    (c) After the domain of f has been restricted as in part (b) find f^{-1}.

13. Solve the followig inequalities. Give your answer in interval notation:

    (a) $x^2 \geq x + 6$

    (b) $(x-1)(x-2)(3-x) < 0$

    (c) $2x^3 + 7x^2 - 15x > 0$