The answers to the second quiz

1. Find all assymptotes for each of the following rational functions:

   (a) $f(x) = \dfrac{2x^2 - 5}{x^2 + 2x - 8}$

   Answer. The denominator is

   $$x^2 + 2x - 8 = (x+4)(x-2)$$

   so the domain of this function is $(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$. None of the roots of the denominator are roots of the numerator so they both determine vertical asymptotes. Thus we have two vertical asymptotes:

   $$x = -4, \quad \text{and} \quad x = 2$$

   Also since the numerator and denominator have the same degree we have a horizontal asymptote:

   $$y = 2$$

   □

   (b) $g(x) = \dfrac{2x^2 - 3x + 5}{x + 1}$

   Answer. The denominator vanishes at $x = -1$ and this is not a root of the numerator. Thus we have a vertical asymptote:

   $$x = -1$$

   Also since the degree of the numerator is one more than the degree of the denominator we have a slant asymptote determined by the quotient of the division. So the slant asymptote is

   $$y = 2x - 5$$

   □

2. Use the properties of logarithms to expand the following expression as much as possible:

   $$\ln \sqrt[3]{\frac{x^2 y}{z^5}}$$

   Answer.

   $$\begin{aligned}
   \ln \sqrt[3]{\frac{x^2 y}{z^5}} &= \ln \left(\frac{x^2 y}{z^5}\right)^{1/3} \\
   &= \frac{1}{3} \ln \left(\frac{x^2 y}{z^5}\right) \\
   &= \frac{1}{3} \left(\ln x^2 y - \ln z^5\right) \\
   &= \frac{1}{3} \left(\ln x^2 + \ln y - \ln z^5\right) \\
   &= \frac{1}{3} \left(2 \ln x + \ln y - 5 \ln z\right) \\
   &= \frac{2 \ln x}{3} + \frac{\ln y}{3} - \frac{5 \ln z}{3}
   \end{aligned}$$

   □

3. Solve the following equation:

$$\log_2(x-4) + \log_2(x+2) = \log_2 7$$

Answer. We have

$$\begin{aligned} \log_2(x-4) + \log_2(x+2) = \log_2 7 &\iff \log_2(x-4)(x+2) = \log_2 7 \\ &\iff \log_2(x^2 - 2x - 8) = \log_2 7 \\ &\implies x^2 - 2x - 8 = 7 \\ &\iff x^2 - 2x - 15 = 0 \\ &\iff (x+3)(x-5) = 0 \\ &\iff x = -3 \quad \text{or} \quad x = 5 \end{aligned}$$

When we plug $x = -3$ in to the original equation we see that it doesn't work since we get $\log_2 -7$ wchich is undefined. $x = 2$ does work. So the only solution is

$$x = 5$$

□

4. Solve the following equation:

$$10^{2x} - 11 \cdot 10^x + 10 = 0$$

Answer. Let $u = 10^x$. Then $10^{2x} = u^2$ and the original equation becomes:

$$u^2 - 11u + 10 = 0$$

Solving this equation we get

$$u = 1 \quad \text{or} \quad u = 10$$

So we have

$$10^x = 1 \quad \text{or} \quad 10^x = 10$$

or equivalently

$$x = 0 \quad \text{or} \quad x = 1$$

□