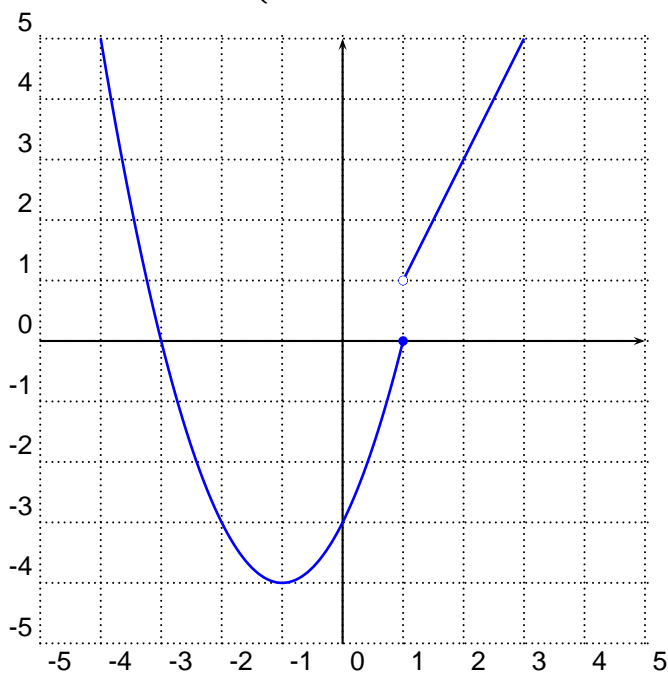**Answers to the first exam for Math 30.**

1. Graph the following function. Possible x or y intercepts should be identified exactly.

$$f(x) = \begin{cases} x^2 + 2x - 3 & \text{if } x \le 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$$

2. Find the domain, the range, and the inverse function of the following function:

$$f(x) = \frac{2}{x+3}$$

Your answers should be in interval notation.

Answer. Domain is $(-\infty, -3) \cup (-3, \infty)$, Range is $(-\infty, 0) \cup (0, \infty)$. The inverse function is

$$f^{-1}(x) = \frac{2}{x} - 3$$

$\square$

3. Verify that the following two functions are inverses of each other: $f(x) = \dfrac{2x - 5}{x + 3}$ and $g(x) = \dfrac{3x + 5}{2 - x}$.

Answer. There are two ways to verify that these two functions are inverses to each other.
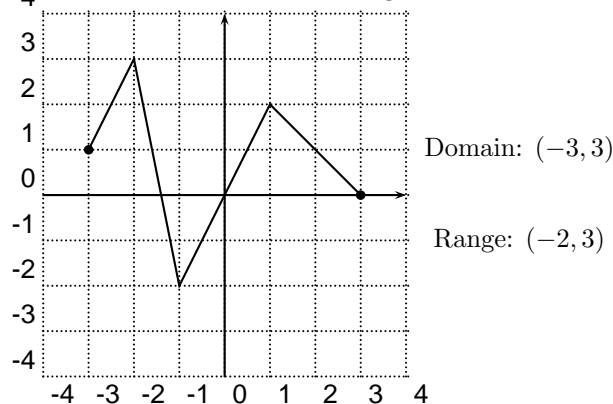
- You can either compute the inverse functions and verify that the g is indeed the inverse of f and f is the inverse of g.

- Or, you can simply verify that,

$$(f \circ g)(x) = x \text{ and } (g \circ f)(x) = x$$

□

4. Find the domain and the range of the function h whose graph is shown:

Domain: $(-3, 3)$

Range: $(-2, 3)$

5. Solve the inequality: $(x + 2)(x - 1)(x - 3) \geq 0$

Answer. The solution set is $[-2, 1] \cup [3, \infty)$.

□

6. Let $f(x) = \sqrt{3x - 6}$ and $g(x) = 2x + 1$. Find the domain and the formula for the function $\dfrac{f}{g}$.

Answer. The formula for $\dfrac{f}{g}$ is

$$\frac{f}{g}(x) = \frac{\sqrt{3x - 6}}{2x + 1}$$

A real number x is in the domain of $\dfrac{f}{g}$ if *both* of the following conditions are satisfied

1. $3x - 6 \geq 0$ or equivalently $x \geq 2$
2. $2x + 1 \neq 0$ or equivalently $x \neq -\dfrac{1}{2}$

Both of these conditions are satisfied in the interval $[2, \infty)$.

□

7. Suppose a ball is thrown directly upward from an initial height of 200 feet with an initial velocity of 96 feet per second. Then the height of the ball (in feet) after t seconds is given by the equation:

$$h(t) = -16t^2 + 96t + 200$$

When will the ball reach its maximum height?

Answer. The graph of h is a parabola with vertex $(3, 344)$. Therefore the ball will reach maximum height after 3 seconds.

□

8. The graph of the function g is obtained by shifting the graph of the function $f(x) = 2x^3$ three units to the right along the x–axis and four units downwards along the y–axis. Find a formula for $g(x)$. (**You don't need to graph** g).

Answer. $g(x) = 2(x-3)^3 - 4$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □