

Fifth Quiz for CSI35
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Directions: This quiz is due Thursday November 16, at 6:00 PM. Please make sure to *justify* all your answers. **No credit will be given for unjustified answers.**

1. The complete graphs K_n for $n \geq 2$ have both Euler and Hamilton circuits. Give an other example of a graph that has both Euler and Hamilton circuits.
2. Count von Diamond has been murdered in his estate. The internationally known detective (and part time graph theorist) Inspector Clouseau has been called in to investigate. The butler claims that he saw the gardener enter the pool room (where the murder took place) and then, shortly after, leave that room by the same door. On the other hand, the gardener says that he cannot be the man that the butler saw because he entered the house, went through each door exactly once and then left the house. Inspector Clouseau checks the floor plan (see Figure 1) and within minutes declares the case solved. Who done it?

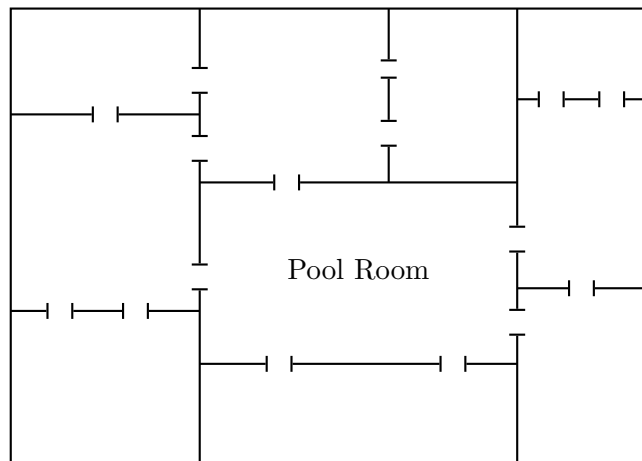


Figure 1: The floor of Von Diamond Estate

3. Find a Hamiltonian path in the Petersen graph (see Figure 2).
4. Use Dijkstra's algorithm to find the shortest path between a and z in the wighted graph of Figure 3.
5. *Extra Credit* Prove that the Petersen graph does not have a Hamilton circuit.

Hint. One way to proceed is the following: Prove first that if there is a Hamilton circuit, say c , then in c there must be an even number of edges connecting the outer pentagon to the inner star. Then proceed case by case: prove that there is no c with two edges connecting the pentagon to the star and then that there is no c that has four such edges either.

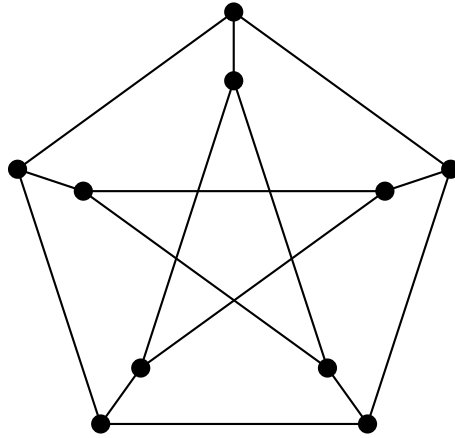


Figure 2: The Petersen graph

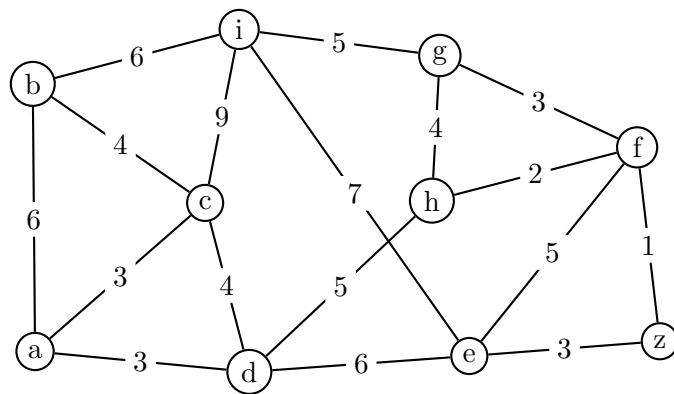


Figure 3: A weighted graph