First Quiz for CSI35

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Directions: This quiz is due Thursday October 5, at 6:00 PM.

1. Alice and Bob play a game by taking turns removing 1, 2 or 3 stones from a pile that initially has n stones. The person that removes the last stone wins the game. Alice plays always first.
   (a) Prove by induction that if n is a multiple of 4 then Bob has a wining strategy.
   (b) Prove that if n is not a multiple of 4 then Alice has a wining strategy.

2. Recall that a *bit string* is a word on the alphabet $\{0, 1\}$. Let O be the function that counts the number of zeros in s.
   (a) Give a recursive definition of $O(s)$.
   (b) Use structural induction to prove that for two string bits s and t we have:
   $$O(st) = O(s) + O(t)$$
   where, st stands for the concatenation of the two strings s and t.

3. For a rooted tree T let $v(T)$ and $e(T)$ denote the number of vertices and the number of edges of T respectively. Use structural induction to prove that for all rooted trees T,
   $$v(T) - e(T) = 1$$

4. Recall the definition of the Fibonacci numbers $\{f_n\}$:
   $$f_n = \begin{cases} 0 & \text{if n=0,} \\ 1 & \text{if n=1,} \\ f_{n-1} + f_{n-2} & \text{otherwise} \end{cases}$$
   Prove that for all positive integers n we have:
   $$f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$$