Midterm exam for CSI35

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Directions: This quiz is due Thursday November 2, at 6:00 PM.

1. What is the *halting problem*? What does Turing's theorem state exactly?
2. Prove that if n is an odd integer then $n^4 \equiv 1 \bmod 4$.
3. Provide a simple formula that generates the terms of the sequence that begins with $15, 8, 1, -6, \ldots$
4. Give a recursive definition of the sequence $\{a_n\}$ if
$$a_n = 5n - 6, \quad \text{for} \quad n = 0, 1, 2, \ldots$$
5. Prove by mathematical induction that for any natural number n we have:
$$2 + 4 + 8 + \cdots + 2n = n(n+1)$$
6. Prove using mathematical induction that a set with n elements has 2^n subsets.
7. Let f_n denote the n-th Fibonacci number where n is a natural number. Show that
$$f_{n-1} f_{n+1} - f_n^2 = (-1)^n$$
8. Recall that a *bit string* is a word in the alphabet $\{0, 1\}$.
   (a) Give a recursive definition of the set of bit strings.
   (b) Let I be the function that counts the number of ones in a bit string. Give a recursive definition of I.
   (c) Use structural induction to prove that
$$I(st) = I(s) + I(t)$$
9. Consider the relation R represented by the matrix
$$M_R = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

(a) Is R reflexive?

(b) Is R symmetric?

(c) Is R transitive?

(d) Draw the digraph representing R.

10. Give the definition of an equivalence relation.

11. Is the following relation
$$R = \{(a, b) | a^2 = b^2\}$$
an equivalence relation? If yes what is the equivalence class of 5?

12. Consider the relation R defined on the set of all positive real numbers as follows:
$$(a, b) \in R \quad \text{iff} \quad \frac{a}{b} \in \mathbb{Q}\,,$$
where $\mathbb{Q}$ stands for the set of rational numbers. Prove that R is an equivalence relation.

13. Let R be the equivalence relation generated by the following partition of $\{1, 2, 3, 4, 5, 6\}$:
$$\{\{1, 2\}, \{3\}, \{4, 5, 6\}\}$$
(a) Find the equivalence classes of R.

(b) Draw the digraph representing R.

14. (**Extra Credit**) List all equivalence relations on the set $A = \{1, 2, 3, 4\}$.