Second Review for Math 13 Fall 2005

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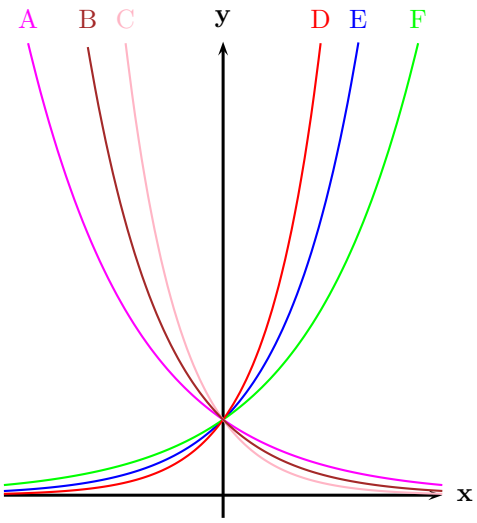
Please note: You should fully justify your answers.

1. Match each graph with a function:



   (a) $f(x) = e^x$,
   (b) $f(x) = e^{-x}$
   (c) $f(x) = 4^x$
   (d) $f(x) = 2^x$
   (e) $f(x) = 4^{-x}$
   (f) $f(x) = \left(\frac{1}{2}\right)^x$

2. Graph each of the following functions:
   (a) $f(x) = \ln x$
   (b) $f(x) = 3\log_2 x$.

3. What is the domain of the following function:

$$f(x) = \ln(\ln x)\,.$$

4. Express as a sum difference or multiple of logarithms:

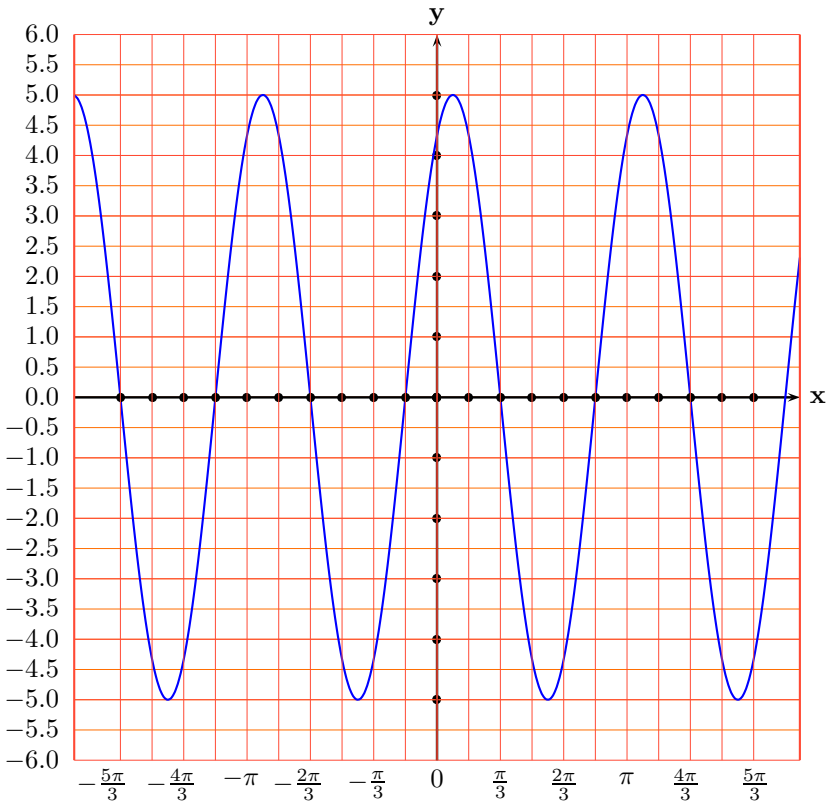
$$\ln\left(\frac{x\sqrt[7]{y}}{z^2w^3}\right)\,.$$

5. Write the following expression as a single logarithm:

$$4\log_4 x - \frac{2}{3}\log_4 2y + 3\log_4 x + 2\,.$$

6. Solve the following equations:
   (a) $\pi^{2x+3} = 17$
   (b) $\frac{1}{2}\log_3(x+3) = 2$
   (c) $\log(x-3) + \log x = 2$
   (d) $2^x - 6\,(2^{-x}) = 1$
   (e) $3^{x-2} = 2^{x+1}$
   (f) $2\log_x 2 + \log_2 x = 3$.

7. Find an equation of the following *sinusoidal* curve:



8. Sketch the graph of the following function:

$$f(x) = -4\cos(x - \frac{\pi}{4})$$

9. Find the function (and graph it) of the form $y = 2\cos(2x - c)$ that passes through the point $(\pi/6, 2)$ for the smallest possible positive value of c.

10. A particle moves on a circle of radius 6 with constant angular velocity $\omega = 4.00\,\text{rad/s}$ and at time $t = 0$ its initial angle is $\pi/4\,\text{rad}$. Write the equation that describes the displacement of the projection of the particle on the y-axis.

11. Sketch the graph of each of the following functions:
   (a) $y = \sec x$

(b) $y = \csc x$

(c) $y = \tan x$

(d) $y = \cot x$.

12. Prove the following identities:

(a) $\csc\theta \sec\theta - \tan\theta = \cot\theta$

(b) $\dfrac{1+\sin x}{\cos x} = \dfrac{\cos x}{1-\sin x}$

(c) $\dfrac{\cos^2 x + 2\sin x - 1}{\cos^2 x + 3\sin x - 3} = \dfrac{1}{1-\csc x}$

(d) $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$