

MTH05X take home exam 1
The answers

1. Evaluate: $3 - 2(5 - 7) - 3^3 \div 9 \cdot 4$.

Answer.

$$\begin{aligned} 3 - 2(5 - 7) - 3^3 \div 9 \cdot 4 &= 3 - 2(5 - 7) - 27 \div 9 \cdot 4 \\ &= 3 - 2(-2) - 27 \div 9 \cdot 4 \\ &= 3 + 4 - 27 \div 9 \cdot 4 \\ &= 3 + 4 - 3 \cdot 4 \\ &= 3 + 4 - 12 \\ &= 7 - 12 \\ &= -5 \end{aligned}$$

□

2. Evaluate: $\frac{-20}{-12} \cdot \frac{-8}{45} \cdot \frac{21}{5} \cdot \frac{-4}{5} \cdot \left(-\frac{6}{7}\right)$.

Answer.

$$\begin{aligned} \frac{-20}{-12} \cdot \frac{-8}{45} \cdot \frac{21}{5} \cdot \frac{-4}{5} \cdot \left(-\frac{6}{7}\right) &= \frac{5}{3} \cdot \frac{8}{45} \cdot \frac{3}{5} \cdot \frac{4}{5} \cdot \frac{6}{1} \\ &= \frac{1}{1} \cdot \frac{8}{15} \cdot \frac{1}{1} \cdot \frac{4}{5} \cdot \frac{2}{1} \\ &= -\frac{64}{75} \end{aligned}$$

□

3. Evaluate, if $a = -\frac{4}{3}$ and $b = \frac{7}{9}$: $-4a - 3b$

Proof.

$$\begin{aligned} -4\left(-\frac{4}{3}\right) - 3\left(\frac{7}{9}\right) &= \frac{16}{3} - \frac{7}{3} \\ &= \frac{9}{3} \\ &= 3 \end{aligned}$$

□

4. Evaluate if $a = -1$, $b = 2$, $c = -4$, and $d = -5$: $(a^2 - b^2)(c - d)$.

Answer.

$$\begin{aligned}((-1)^2 - (2)^2)((-4) - (-5)) &= (1 - 4)(-4 + 5) \\ &= (-3)(1) \\ &= -3\end{aligned}$$

□

5. Solve the equation:

$$5(2x - 1) + 6 = 3x - 20$$

Answer.

$$\begin{aligned}5(2x - 1) + 6 = 3x - 20 &\iff 10x - 5 + 6 = 3x - 20 \\ &\iff 10x + 1 = 3x - 20 \\ &\iff 10x - 3x = -20 - 1 \\ &\iff 7x = -21 \\ &\iff x = -\frac{21}{7} \\ &\iff x = -3\end{aligned}$$

□

6. Solve the equation:

$$x - 2(4 - 3x) + 5 = 5x + 3(x - 1) - 1$$

Answer.

$$\begin{aligned}x - 2(4 - 3x) + 5 = 5x + 3(x - 1) - 1 &\iff x - 8 + 6x + 5 = 5x + 3x - 3 - 1 \\ &\iff 7x - 3 = 8x - 4 \\ &\iff 4 - 3 = 8x - 7x \\ &\iff 1 = x\end{aligned}$$

□

7. Solve the equation:

$$\frac{x + 8}{2} + \frac{x + 5}{3} = x + 7$$

Answer.

$$\begin{aligned}\frac{x + 8}{2} + \frac{x + 5}{3} = x + 7 &\iff 6 \cdot \frac{x + 8}{2} + 6 \cdot \frac{x + 5}{3} = 6 \cdot (x + 7) \\ &\iff 3(x + 8) + 2(x + 5) = 6x + 42 \\ &\iff 3x + 24 + 2x + 10 = 6x + 42 \\ &\iff 5x + 34 = 6x + 42 \\ &\iff 34 - 42 = 6x - 5x \\ &\iff -8 = x\end{aligned}$$

□

8. Solve for y : $6x - 2y = 4$.

Answer.

$$\begin{aligned}
6x - 2y = 4 &\iff -2y = 4 - 6x \\
&\iff y = \frac{4 - 6x}{-2} \\
&\iff y = \frac{4}{-2} - \frac{6x}{-2} \\
&\iff y = -2 + 3x
\end{aligned}$$

□

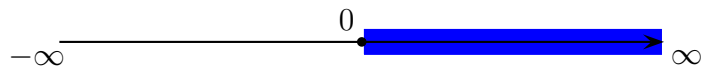
9. Solve the following inequality, give the answer using interval notation and graph the solution set.

$$3 - (4x - 2) \leq 5 - x$$

Answer.

$$\begin{aligned}
3 - (4x - 2) \leq 5 - x &\iff 3 - 4x + 2 \leq 5 - x \\
&\iff -4x + 5 \leq 5 - x \\
&\iff -4x + x \leq 5 - 5 \\
&\iff -3x \leq 0 \\
&\iff x \geq 0
\end{aligned}$$

The graph of the solution set in the number line is:



In interval notation the solution is $[0, \infty)$

□

10. Find five solutions to the following equation: $2x - y = 6$.

Answer. We choose values for one variable and solve for the other.

(a) $x = 0$:

$$2 \cdot 0 - y = 6 \iff -y = 6 \iff y = -6$$

So we get the solution $(0, -6)$

(b) $y = 0$:

$$2x - 0 = 6 \iff 2x = 6 \iff x = 3$$

So we get the solution $(3, 0)$

(c) $x = 1$:

$$2 \cdot 1 - y = 6 \iff 2 - y = 6 \iff y = -4$$

So we get the solution $(1, -4)$

(d) $x = -1$:

$$2 \cdot (-1) - y = 6 \iff -2 - y = 6 \iff y = -8$$

So we get the solution $(-1, -8)$

(e) $x = 2$:

$$2 \cdot 2 - y = 6 \iff 4 - y = 6 \iff y = -2$$

So we get the solution $(2, -2)$

□

11. Complete the following pairs: $(-1, \quad)$, $(3, \quad)$, $(-4, \quad)$, $(\quad, \frac{1}{2})$, $(\quad, -\frac{3}{4})$, $(\quad, \frac{2}{3})$ to solutions of the equation:

$$-x + 2y = 9$$

Answer. $(-1, 4)$, $(3, 6)$, $(-4, \frac{13}{2})$, $(-8, \frac{1}{2})$, $(-\frac{21}{2}, -\frac{3}{4})$, $(-\frac{23}{3}, \frac{2}{3})$

□

12. Twice the sum of two consecutive even integers is 20. Find the two integers.

Answer. Let n be the smaller of the two consecutive even integers. Then the larger will be $n + 2$ and the sum of the two integers will be $n + (n + 2)$. So we have the equation

$$\begin{aligned} 2(n + (n + 2)) = 20 &\iff 2(2n + 2) = 20 \\ &\iff 4n + 4 = 20 \\ &\iff 4n = 20 - 4 \\ &\iff 4n = 16 \\ &\iff n = \frac{16}{4} \\ &\iff n = 4 \end{aligned}$$

So the smaller of the two consecutive even integers is 4. It follows that the other integer is 6.

□

13. I have some nickels, dimes, and quarters in my pockets. I have 3 less nickels than dimes, while the number of quarters is 4 less than twice the number of dimes. If the total value of the change in my pocket is \$1.45 how many nickels do I have in my pocket?

Answer. Let x be the number of dimes. Then the number of nickels is $x - 3$ and the number of quarters is $2x - 4$. We can summarize the given information in the following table.

Coin	Quantity	Value per Unit	Total Value
Nickel	$x - 3$	5	$5(x - 3)$
Dime	x	10	$10x$
Quarter	$2x - 4$	25	$25(2x - 4)$
Total			145

So we have the following equation:

$$\begin{aligned}
5(x - 3) + 10x + 25(2x - 4) = 145 &\iff 5x - 15 + 10x + 50x - 100 = 145 \\
&\iff 65x - 115 = 145 \\
&\iff 65x = 145 + 115 \\
&\iff 65x = 260 \\
&\iff x = \frac{260}{65} \\
&\iff x = 4
\end{aligned}$$

Therefore I have 4 dimes in my pocket; since I have 3 less nickels than dimes it follows that I have one nickel in my pocket. \square

14. The length of a rectangle is 5 cm less than 3 times its width. If the perimeter of the rectangle is 38 cm find its dimensions.

Answer. Let x be the length of the rectangle in centimetres. Then its width will be $3x - 5$. The perimeter of the rectangle will then be $2x + 2(3x - 5)$ and we have the equation:

$$\begin{aligned}
2x + 2(3x - 5) = 38 &\iff 2x + 6x - 10 = 38 \\
&\iff 8x - 10 = 38 \\
&\iff 8x = 38 + 10 \\
&\iff 8x = 48 \\
&\iff x = \frac{48}{8} \\
&\iff x = 6
\end{aligned}$$

So the length of the rectangle is 6 cm. It follows that its width is $3 \cdot 6 - 5 = 13$ centimetres. \square